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Integration of finite element and multibody system algorithms for the analysis of human body motion

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Abstract

The objective of this paper is to demonstrate the use of the absolute nodal coordinate formulation (ANCF) in the integration of finite element (FE) and multibody system (MBS) algorithms for modeling the rigid body contact and the ligament flexibility in bio-mechanics applications. To this end, a general formulation based on ANCF finite elements for modeling the contact in bio-mechanics applications is presented. Each contact surface is described in a parametric form using two surface parameters that enter into the ANCF finite element geometric description. A set of nonlinear algebraic equations that depend on the surface parameters are developed. These nonlinear algebraic equations are solved iteratively in order to determine the location of the contact points. This formulation is implemented in a general MBS algorithm that allows for modeling rigid and flexible bodies. ANCF finite elements can also be used to describe the large displacement of the ligaments, muscles, and soft tissues (LMST). The computational algorithm developed in this investigation can be demonstrated using a knee joint model in which the ACL and PCL are modeled using linear spring damper elements and the LCL and MCL are modeled using the large displacement ANCF finite elements that allows for using general constitutive models and capture the deformation of the ligament cross sections.

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1. Introduction

The study of the human body motion as a multibody system is a challenging research field that has witnessed significant developments over the last years. In general, most of the investigations focused on

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the simulation of human tasks are based on the assumption that the joints that constrain the relative motion of the system components are ideal or perfect joints. In order to better understand the performance of human body biomechanics, it is necessary to develop realistic and detailed models that more accurately describe the characteristics of the human joints; an important example of which is the knee joints [1].

In recent years, most of the studies have approximated the human knee joint using images of cadaveric knees [2]. It was previously shown that the large displacement knee joint mechanics can be examined using MBS algorithms and ANCF finite elements [3]. ANCF finite elements were used to model the large displacement of the LCL and MCL. It was shown that ANCF finite elements that lead to constant mass matrix can capture the cross section deformations of the ligaments and allow for the developments of more general insertion site models. Nonetheless, previous studies by the authors did not consider the geometry of the contact between the femur and the tibia of the knee joint. The knee joint, shown in Fig.1, has three surfaces, covered by articular cartilage, separated by two menisci and several ligaments: lateral collateral ligament (LCL), medial collateral ligament (MCL) and the anterior and posterior cruciate ligaments (ACL, PCL) stabilize the joint. Understanding the biomechanics of the knee joint is important in the diagnosis of injuries and in making surgical decisions [4], [5]. Realistic knee joint models, however, requires a successful integration of large displacement FE and MBS algorithms [3].

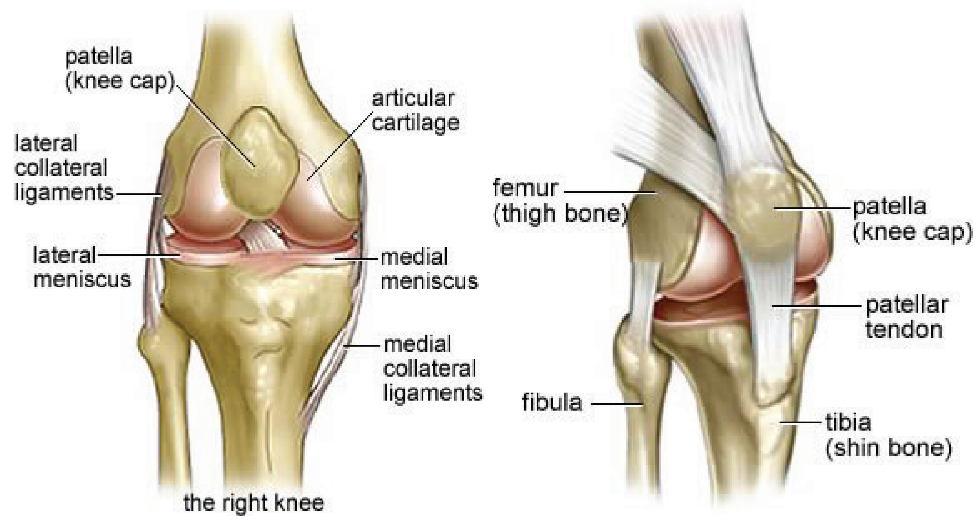

Fig.1 Anatomy of the knee (http://www.aclsolutions.com/anatomy.php)

Ligaments, muscles, and soft tissues (LMST) experience large displacements that can be accurately represented using nonlinear FE formulations. For example, the knee joint LCL and MCL structural flexibility can be accurately modeled, as demonstrated in previous publications, using ANCF finite elements. This is the approach that will be employed in this investigation. In addition to using ANCF finite elements in modeling the ligament deformation, ANCF elements are also used to model the rigid body contact in the knee joint model. In this study, the bones of the knee joint are assumed to be rigid and the tibial and femoral condyle surfaces are represented using parametric ANCF geometry that can be converted to B-spline or NURBS representations; thereby allowing for the integration of CAD, FE and MBS algorithms. In the femur/tibia elastic contact formulation used in this study, the femur is assumed to have six degrees of freedom with respect to the tibia; small penetrations at the contact points are allowed. Using this approach, a compliant force element that employs stiffness and damping forces is used to determine the normal contact force. The location of the contact points are determined by solving a set of nonlinear algebraic equations; for each contact four algebraic equations are solved to determine the four parameters that describe the geometry of the femur and the tibia surfaces. When a contact occurs, the normal contact force is determined using the aforementioned compliant force model that defines the generalized contact forces that enter into the formulation of the system equations of motion [6].

2. Contact geometry

Two steps are employed in the computational algorithm used to obtain the numerical solution of the femur/tibia contact problem. The first is the geometry step, in which the locations of the points of contact between the femur and tibia are determined. The second step is the force calculation step. In this second step, the forces that act on the femur and tibia as a result of the contact are determined. The accuracy of the numerical solution of the contact problem depends strongly on the accurate prediction of the location of the contact points. The solution for the contact locations requires an accurate representation of the geometry of the femur and tibia surfaces. In this study, ANCF finite elements are used to describe the geometry of the femur and tibia surfaces. In the absolute nodal coordinate formulation, the global position vector $\mathbf{r}^{ij}$ of an arbitrary point on the fully parameterized finite element j of body i can be defined using the element shape function and the vector of nodal coordinates as $\mathbf{r}^{ij} = \mathbf{S}^{ij}(x,y,z)\mathbf{e}^{ij}(t)$, where $\mathbf{S}^{ij}$ is the element shape function matrix expressed in terms of the element local coordinates x, y, and z, t is time, and $\mathbf{e}^{ij}$ is the vector of the element nodal coordinates. It has been shown in the literature that Bezier and B-spline representations used in CAD modelling can be converted to ANCF geometry in a straight forward manner using a linear transformation [7]. This fact allows for establishing a simple interface between CAD systems and the FE/MBS analysis software.

A complete parameterization of the surfaces is used in this investigation in order to accurately determine the location of the point of contact between two bodies. A set of four surface parameters can be used to describe the geometry of the two surfaces in contact. The surface parameters can be written in a vector form as

$$\mathbf{s} = \begin{bmatrix} s_1^i & s_2^i & s_1^j & s_2^j \end{bmatrix}^T \tag{1}$$

where superscripts i and j refer to bodies i (femur) and j (tibia), respectively. Using these parameters, the location of the contact point P can be defined, respectively, in the coordinate systems of bodies i and j as

$$\bar{\mathbf{u}}_P^i\left(s_1^i, s_2^i\right) = \begin{bmatrix} x^i\left(s_1^i, s_2^i\right) \\ y^i\left(s_1^i, s_2^i\right) \\ z^i\left(s_1^i, s_2^i\right) \end{bmatrix}, \qquad \bar{\mathbf{u}}_P^j\left(s_1^j, s_2^j\right) = \begin{bmatrix} x^j\left(s_1^j, s_2^j\right) \\ y^j\left(s_1^j, s_2^j\right) \\ z^j\left(s_1^j, s_2^j\right) \end{bmatrix} \tag{2}$$

The tangents to the surface at the contact point are defined in the body i coordinate system as

$$\bar{\mathbf{t}}_1^k = \frac{\partial \bar{\mathbf{u}}_P^k}{\partial s_1^k}, \qquad \bar{\mathbf{t}}_2^k = \frac{\partial \bar{\mathbf{u}}_P^k}{\partial s_2^k}, \quad k = i, j \tag{3}$$

Using these tangent vectors, the normal vector can be defined as $\bar{\mathbf{n}}^k = \bar{\mathbf{t}}_1^k \times \bar{\mathbf{t}}_2^k$. The parameterization used in Eq. 2 for the surfaces, as well as the tangent and the normal vectors, can be used to describe the geometry of the femur and tibia surfaces. This parameterization allows for the description of general surfaces and also allow for the use of numerical or tabulated data to define the surface geometry.

3. Contact geometry and forces

In this section, the method used to define the location of the femur/tibia contact points online is first described. This geometric contact method is used to define the parameters that enter into the formulation of the contact forces. These contact forces are used to define the generalized contact forces that enter into the formulation of the equations of motion of the knee joint model.

3.1 Background

Since the knee is statically indeterminate [8] the ideal computational environment would combine a MBS model to predict ligament and muscle forces with a deformable contact model of the articular surface geometry to predict contact pressures. Bei and Fregly presented a methodology for simulating deformable contact in human joints within an MBS environment [9]. The approach requires use of specially prepared contact surfaces and efficient distance calculation methods using a contact solver selected for its applicability to human joints. The methodology was successfully applied to static analysis via dynamic simulation of a natural knee contact model created from MRI. This contact model possesses several important limitations; the materials are assumed to be isotropic and homogenous, and the model does not take into account the effect of menisci [9].An additional study provided a simplified model of the human knee joint for studying tibio-femoral contact behavior assuming rigid bones indicated that this assumption is valid when the contact behavior of cartilaginous joints are of interest [2].

In previous publications [10], [3], [11], the sliding between the femur and tibia was modelled by using a kinematic revolute joint placed at a specific location in the model with only the two outside ligaments (MCL, LCL) providing stability. This joint allows only rotation of the femur relative to the tibia in the sagittal plane. One goal of the current study is to improve the kinematic and force knee joint model by allowing more degrees of freedom of the femur with respect to the tibia and include the two interior cruciate ligaments (ACL and PCL). Having additional degrees of freedom requires the use of a femur/tibia contact force model that was not required when the kinematic revolute joint was used in previous investigations. The model in this investigation is based on the assumption that the outer surface of each condyle of the femur can be simulated by a curved rigid surface using ANCF finite elements.

There are two different methods, the constraint and elastic contact approaches that are commonly used to solve MBS contact problems. These two approaches lead to different mathematical models for determining the normal contact force. In the constraint approach, the non-conformal contact conditions are imposed on the motion of the system, and the normal contact force is predicted as a constraint force obtained using the technique of Lagrange multipliers. In this case, no separations or penetrations between tibia and femur are allowed since rigid body contact assumptions are used.

3.2 Femur/Tibia elastic contact formulation

In the elastic approach, on the other hand, no contact constraint conditions are imposed and small penetrations at the contact points are allowed. The location of the contact points is determined by first solving a set of algebraic equations to determine the vector of surface parameter and using this data to determine the penetration. For each contact, four algebraic equations are solved to determine the four parameters that describe the geometry of the femur and tibia surfaces. These four equations can be written as

$$\left.\begin{array}{l}\mathbf{t}_1^{\mathrm{t}} \times \left(\mathbf{r}^{\mathrm{f}} - \mathbf{r}^{\mathrm{t}}\right) = 0, \quad \mathbf{t}_2^{\mathrm{t}} \times \left(\mathbf{r}^{\mathrm{f}} - \mathbf{r}^{\mathrm{t}}\right) = 0 \\ \mathbf{t}_1^{\mathrm{f}} \times \mathbf{n}^{\mathrm{t}} = 0, \quad \mathbf{t}_2^{\mathrm{f}} \times \mathbf{n}^{\mathrm{t}} = 0\end{array}\right\} \tag{4}$$

where $\mathbf{t}_1^i$ and $\mathbf{t}_2^i$ ($i= t, f$), are respectively, the tangents to the tibia and femur surfaces at the potential contact point, $\mathbf{r}^{ft}=\mathbf{r}^f - \mathbf{r}^t$ is the vector that defines the relative position of the point on the femur with respect to the point on the tibia; and $\mathbf{n}^t$ is the normal to the tibia surface at the potential contact point. Because the tangent and the normal vectors are functions of the surface parameters, and assuming that the generalized coordinates of the femur and tibia are known, one can write the set of algebraic equations of Eq. 4 in a vector form as **E(s) = 0**, where **E** is the vector of nonlinear algebraic equations that can be solved using an iterative Newton-Raphson algorithm for the surface parameters that define the potential non-conformal contact points. This requires evaluating the Jacobian matrix of the algebraic equations and iteratively solving the following system for each contact to determine the Newton differences associated with the surface parameters:

$$\begin{bmatrix} \mathbf{t}_1^t \cdot \mathbf{t}_1^f & \mathbf{t}_1^t \cdot \mathbf{t}_2^f & \dfrac{\partial \mathbf{t}_1^t}{\partial s_1^t} \cdot \mathbf{r}^{ft} - \mathbf{t}_1^t \cdot \mathbf{t}_1^t & \dfrac{\partial \mathbf{t}_1^t}{\partial s_2^t} \cdot \mathbf{r}^{ft} - \mathbf{t}_1^t \cdot \mathbf{t}_2^t \\ \mathbf{t}_2^t \cdot \mathbf{t}_1^f & \mathbf{t}_2^t \cdot \mathbf{t}_2^f & \dfrac{\partial \mathbf{t}_2^t}{\partial s_1^t} \cdot \mathbf{r}^{ft} - \mathbf{t}_2^t \cdot \mathbf{t}_1^t & \dfrac{\partial \mathbf{t}_2^t}{\partial s_2^t} \cdot \mathbf{r}^{ft} - \mathbf{t}_2^t \cdot \mathbf{t}_2^t \\ \dfrac{\partial \mathbf{t}_1^f}{\partial s_1^f} \cdot \mathbf{n}^t & \dfrac{\partial \mathbf{t}_1^f}{\partial s_2^f} \cdot \mathbf{n}^t & \dfrac{\partial \mathbf{n}^t}{\partial s_1^f} \cdot \mathbf{t}_1^f & \dfrac{\partial \mathbf{n}^t}{\partial s_2^f} \cdot \mathbf{t}_1^f \\ \dfrac{\partial \mathbf{t}_2^f}{\partial s_1^f} \cdot \mathbf{n}^t & \dfrac{\partial \mathbf{t}_2^f}{\partial s_2^f} \cdot \mathbf{n}^t & \dfrac{\partial \mathbf{n}^t}{\partial s_1^f} \cdot \mathbf{t}_2^f & \dfrac{\partial \mathbf{n}^t}{\partial s_2^f} \cdot \mathbf{t}_2^f \end{bmatrix} \begin{bmatrix} \Delta s_1^f \\ \Delta s_2^f \\ \Delta s_1^t \\ \Delta s_2^t \end{bmatrix} = - \begin{bmatrix} \mathbf{t}_1^t \cdot \mathbf{r}^{ft} \\ \mathbf{t}_2^t \cdot \mathbf{r}^{ft} \\ \mathbf{t}_1^f \cdot \mathbf{n}^t \\ \mathbf{t}_2^f \cdot \mathbf{n}^t \end{bmatrix} \tag{5}$$

In this equation, Δs_1^f, Δs_2^f, Δs_1^t, Δs_2^t are the Newton differences. Convergence is achieved when the norm of the violation of the algebraic equations or the norm of Newton differences is less than a specified tolerance. Having determined the vector of the surface parameters, the penetration δ can be calculated as $\delta=\mathbf{r}^{ft} \cdot \mathbf{n}^t$. Knowing the penetration and its time derivative, the normal contact force can be evaluated using the following equation:

$$F = -K_h \delta^{3/2} - C\dot{\delta}\left|\delta\right| \tag{6}$$

where K_h is the Hertzian constant that depends on the surface curvatures and the elastic properties and C is an assumed damping coefficient. The time rate of penetration, $\dot{\delta}$, can be evaluated as the dot product of the relative velocity vector between the contact points on the femur and on the tibia and the normal vector to the surface at the contact point. The absolute value of the penetration, $\left|\delta\right|$, is introduced in the preceding equation in order to guarantee that the contact force is zero when the penetration is zero.

4. Ligaments modeling

The necessary knee joint stability for optimal daily function is provided by the interaction of various articulations, menisci, ligaments as well as muscle forces [12]. Ligaments are connective tissue that connects bones to other bones, and are an important part of knee anatomy. Tendons and ligaments display time- and history-dependent viscoelastic properties that reflect the complex interactions of collagen and the surrounding proteins and ground substance. The predominant kinematic characteristics of the knee are determined by the curvatures of the femoral and tibial articulating surfaces as well as by the orientation of the knee [13].

The ligaments control the passive motion of the knee joint while the dynamic stability of the joint is provided by muscular movements. The tibiofemoral joint is supported by the medial collateral ligament (MCL), lateral collateral ligament (LCL), anterior cruciate ligament (ACL) and posterior cruciate ligament (PCL). They are the passive load-carrying structures of the joint and serve as a backup to the muscles [14]. These ligaments assist in maintaining the relative position of the tibia and femur so that contact is appropriate and at the right time. The pair of collateral ligaments maintains the internal stability while the cruciate ligaments allow the tibia to move in the sagittal plane relative to the femur. Working together, the four ligaments are the most important structures in controlling stability of the knee. As was mentioned previously this study is a continuation of previous publications where only two of the ligaments MCL and LCL were modelled using ANCF finite elements. Because the MCL and LCL were considered by the authors in previous publications, this study will extend the MCL and LCL model to include the tibiofemoral contact and the two cruciate ligaments.

4.1 Anterior cruciate ligament (ACL)

The ACL is one of the four major ligaments of the knee. It provides primary restraint to anterior displacement of the tibia as well as rotational stability. The kinematics of the ACL has received much attention because of its important role in normal knee function as well as in ligament reconstruction [15]. The ligament is the primary restraint against anterior tibial displacement and internal rotation of the tibia at the knee. Non-contact ACL injuries occur when a high force at the joint in the direction of either internal rotation or anterior tibial translation exceeds the tensile strength of the ligament [16]. The ACL is modelled in this study using a linear spring damper element with a length of 4.031 cm, stiffness of 242000 N/m [17].

4.2 Posterior cruciate ligament (PCL)

This ligament, which connects the tibia to the femur, is located at centre of the knee, behind ACL. The PCL is reported to be stronger than the ACL, and is not injured as often as the ACL [18]. The configuration of the PCL allows the ligament to resist forces pushing the tibia posterior relative to the femur. The function of the PCL is to prevent the femur from sliding off the anterior edge of the tibia and to prevent the tibia from displacing posterior to the femur. The PCL is modelled in this study using a linear spring damper element with a length of 3.033 cm, stiffness of 200000 N/m [18].

5. Multibody system equations

The nonlinear FE formulation can be integrated with the computational MBS algorithms that are designed to solve the differential and the algebraic equations of complex systems. In order to be able to solve the resulting nonlinear dynamic equations, a multi-formulation approach will be used. One can use either the floating frame of reference formulation or rigid body formulation to model the tibia, fibula and femur. In the current work all the bones are modelled as rigid bodies. The floating frame of reference formulation employs coupled reference and elastic coordinates. In this investigation the flexible bodies (ligaments) will be modelled using the large displacement finite element absolute nodal coordinate formulation. Joint constraints that describe insertion site kinematics and specified motion trajectories are being formulated using a set of nonlinear algebraic constraint equations that are adjoined to the system differential equations of motion using the technique of Lagrange multipliers. The following augmented form of the equations of motion is used to obtain the vector of reference, elastic, and absolute accelerations [19]:

$$\begin{bmatrix} \mathbf{m}_{rr} & \mathbf{m}_{rf} & \mathbf{0} & \mathbf{C}_{\mathbf{q}_r}^{\mathrm{T}} \\ \mathbf{m}_{fr} & \mathbf{m}_{ff} & \mathbf{0} & \mathbf{C}_{\mathbf{q}_f}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} & \mathbf{m}_{aa} & \mathbf{C}_{\mathbf{q}_a}^{\mathrm{T}} \\ \mathbf{C}_{\mathbf{q}_r} & \mathbf{C}_{\mathbf{q}_f} & \mathbf{C}_{\mathbf{q}_a} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_f \\ \ddot{\mathbf{q}}_a \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_r \\ \mathbf{Q}_f \\ \mathbf{Q}_a \\ \mathbf{Q}_c \end{bmatrix} \tag{7}$$

In this equation, subscripts r, f, and a refer respectively, to reference, elastic and absolute nodal coordinates; $\mathbf{m}_{rr}$, $\mathbf{m}_{rf}$, $\mathbf{m}_{fr}$, $\mathbf{m}_{ff}$ are the inertia sub-matrices that appear in the floating frame of reference formulation; $\mathbf{m}_{aa}$ is the constant symmetric mass matrix associated with the absolute nodal coordinate formulation which will be an identity matrix when Cholesky coordinates are used [6]; $\mathbf{C}_{\mathbf{q}_r}$, $\mathbf{C}_{\mathbf{q}_f}$, and $\mathbf{C}_{\mathbf{q}_a}$ are the Jacobians of the constraint equations associated, respectively, with the reference, elastic, and ANCF coordinates, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers, and $\mathbf{Q}_c$ is the quadratic velocity vector that results from the differentiation of the constraint equations twice with respect to time. The generalized coordinates, $\mathbf{q}_r$ and $\mathbf{q}_f$, are the coordinates used in the floating frame of reference formulation to describe the motion of rigid and flexible bodies that experience small deformations. The vector $\mathbf{q}_a$ is the vector of ANCF coordinates used to describe the motion of flexible bodies that may undergo large displacement, deformations, and change in the cross section. Knowing the independent coordinates, the nonlinear kinematic constraint equations can be solved for the dependent coordinates using an iterative Newton-Raphson algorithm. Knowing all the coordinates, the dependent velocities can be determined using the algebraic constraint equations at the velocity level.

6. Summary

Previous studies by the authors did not consider the geometry of the contact between the femur and the tibia of the knee joint. A general formulation based on ANCF finite elements for modelling the contact in bio-mechanics applications is presented. Each contact surface is described in a parametric form using two surface parameters that enter into the ANCF finite element geometric description. A set of nonlinear algebraic equations that depend on the surface parameters are developed. Nonlinear algebraic equations are solved iteratively in order to determine the location of the contact points. This formulation is implemented in a general MBS algorithm that allows for modelling rigid and flexible bodies. The computational algorithm developed in this investigation will be demonstrated in future publications using a knee joint model.

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